CHAPTER 9

Compressible Flow

9.1 $c_p = 0.24 \frac{\text{Btu}}{\text{lbm-}^\circ\text{R}} \; 778 \frac{\text{ft-lb}}{\text{Btu}} \; 32.2 \frac{\text{lbm}}{\text{slug}} = \underline{6012 \; \frac{\text{ft-lb}}{\text{slug-}^\circ\text{R}}}$

$$c_v = c_p - R = 6012 - 1716 = \underline{4296 \; \frac{\text{ft-lb}}{\text{slug-}^\circ\text{R}}} = 4296 \; \frac{\text{ft-lb}}{\text{slug-}^\circ\text{R}} \frac{1}{778} \frac{\text{Btu}}{\text{ft-lb}} \frac{1}{32.2} \frac{\text{slug}}{\text{lbm}}$$

$$= \underline{0.171 \; \frac{\text{Btu}}{\text{lbm-}^\circ\text{R}}}$$

9.2 $c_p = c_v + R. \qquad c_p = kc_v. \qquad \therefore c_p = \frac{c_p}{k} + R \;\text{ or }\; c_p\left(1 - \frac{1}{k}\right) = R.$

$$\therefore \underline{c_p = Rk\,/\,(k-1)}.$$

9.3 If $\Delta s = 0$, Eq. 9.1.9 can be written as

$$c_p \ell n \frac{T_2}{T_1} = R \ell n \frac{p_2}{p_1} \quad \text{or} \quad \ell n\left(\frac{T_2}{T_1}\right)^{c_p} = \ell n\left(\frac{p_2}{p_1}\right)^{R}$$

It follows that, using $c_p = c_v + R$ and $c_p / c_v = k$,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{R/c_p} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{k}}.$$

Using Eq. 9.1.7,

$$\frac{T_2}{T_1} = \frac{p_2 r_1}{r_2 p_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{k}} \quad \text{or} \quad \frac{r_1}{r_2} = \left(\frac{p_2}{p_1}\right)^{-1/k}.$$

Finally, this can be written as

$$\frac{p_2}{p_1} = \left(\frac{r_2}{r_1}\right)^{k}.$$

9.4 Substitute Eq. 4.5.18 into Eq. 4.5.17 and neglect potential energy change:

$$\frac{\dot{Q} - \dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + \frac{p_2}{r_2} - \frac{p_1}{r_1} + \tilde{u}_2 - \tilde{u}_1.$$

Enthalpy is defined in Thermodynamics as $h = \tilde{u} + pv = \tilde{u} + p\ /\ r$. Therefore,

$$\frac{\dot{Q} - \dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + h_2 - h_1.$$

Assume the fluid is an ideal gas with constant specific heat so that $\Delta h = c_p \Delta T.$ Then

$$\frac{\dot{Q} - \dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + c_p\left(T_2 - T_1\right).$$

Next, let $c_p = c_v + R$ and $k = c_p / c_v$ so that $c_p / R = k(k-1)$. Then, with the ideal gas law $T = p\ /\ Rr$, the first law takes the form

$$\frac{\dot{Q} - \dot{W}_S}{\dot{m}} = \frac{V_2^2 - V_1^2}{2} + \frac{k}{k-1}\left(\frac{p_2}{r_2} - \frac{p_1}{r_1}\right).$$

9.5 Differentiate $pr^{-k} = c$ using $d(xy) = ydx + xdy$:

$$r^{-k}dp - pkr^{-k-1}dr = 0.$$

Rewrite:

$$\frac{dp}{dr} = k\frac{p}{r}.$$

9.6 The speed of sound is given by

$$c = \sqrt{dp\ /\ dr}.$$

For an isothermal process $TR = p\ /\ r = K,$ where K is a constant. This can be differentiated:

$$dp = Kdr = RTdr.$$

Hence, the speed of sound is

$$c = \sqrt{RT}.$$

9.7 Eq. 9.1.4 with $\dot{Q} = \dot{W}_S = 0$ is: $\dfrac{V^2}{2} + c_p T = \text{cons't}.$

$$\frac{V^2}{2} + c_p T = \frac{(V + \Delta V)^2}{2} + c_p(T + \Delta T) = \frac{V^2 + 2V\Delta V + (\Delta V)^2}{2} + c_p T + c_p \Delta T.$$

$$\therefore 0 = \frac{2V\Delta V}{2} + \cancel{\frac{(\Delta V)^2}{2}} + c_p \Delta T. \quad \therefore -V\Delta V = c_p \Delta T = \Delta h.$$

We neglected $(\Delta V)^2$. The velocity of a small wave is $V = c.$ $\therefore \underline{\Delta h = -c\Delta V}.$

9.8 For water

$$r\frac{dp}{dr} = 2110 \times 10^6 \text{ Pa}$$

Since $r = 1000 \text{ kg / m}^3,$ we see that

$$c = \sqrt{dp\ /\ dr}$$
$$= \sqrt{2110 \times 10^6\ /\ 1000} = \underline{1453\ \text{m}\ /\ \text{s}}$$

9.9 For water $c = \sqrt{\dfrac{\Delta p}{\Delta r}} \cong \sqrt{\dfrac{dp}{dr}} = \sqrt{\dfrac{2110 \times 10^6}{1000}} = 1453\ \text{m}\ /\ \text{s}.$

$$L = \text{velocity} \times \text{time} = 1453 \times 0.6 = \underline{872\ \text{m}}.$$

9.10 Since $c = 1450$ m/s for the small wave, the time increment is

$$\Delta t = \frac{d}{c} = \frac{10}{1450} = \underline{0.0069\ \text{seconds}}$$

9.11 a) $\text{M} = \dfrac{V}{c} = \dfrac{200}{\sqrt{1.4 \times 287 \times 288}} = \underline{0.588}.$

b) $\text{M} = 600/\sqrt{1.4 \times 1716 \times 466} = \underline{0.567}.$

c) $\text{M} = 200\ /\ \sqrt{1.4 \times 287 \times 223} = \underline{0.668}.$

d) $\text{M} = 600/\sqrt{1.4 \times 1716 \times 392} = \underline{0.618}.$

e) $\text{M} = 200\ /\ \sqrt{1.4 \times 287 \times 238} = \underline{0.647}.$

9.12 $c = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 263} = 256\ \text{m}\ /\ \text{s}. \quad \therefore d = ct = 256 \times 1.21 = \underline{309\ \text{m}}.$

9.13 a) Assume T = 20°C:

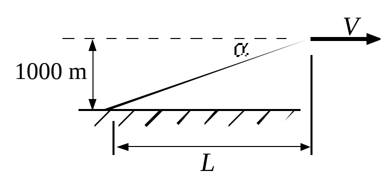
$$c = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 293} = 343\ \text{m}\ /\ \text{s}.$$
$$d = c\Delta t = 343 \times 2 = \underline{686\ \text{m}}$$

b) Assume T = 70°F:

$$c = \sqrt{kRT} = \sqrt{1.4 \times 1716 \times 530} = 1130\ \text{fps}.$$
$$d = c\Delta t = 1130 \times 2 = \underline{2260\ \text{ft}}.$$

For every second that passes, the lightning flashed about 1000 ft away. Count 5 seconds and it is approximately one mile away.

9.14 $c = \sqrt{1.4 \times 287 \times 263} = 256\ \text{m}\ /\ \text{s}. \quad \sin a = \dfrac{1}{M} = \dfrac{c}{V}.$



$\sin a = 0.256. \quad \therefore \tan a = 0.2648 = \dfrac{1000}{L}. \quad \therefore L = 3776\ \text{m}$

$$\Delta t = \frac{3776}{1000} = \underline{3.776\ \text{s}}.$$

9.15 Use Eq. 9.2.13:

a) $\frac{c}{V} = \sin a$ or $V = \frac{\sqrt{1.4 \times 287 \times 288}}{\sin 22^\circ} = \underline{908 \text{ m / s}}$

b) $\frac{c}{V} = \sin a$ or $V = \frac{\sqrt{1.4 \times 1716 \times 519}}{\sin 22^\circ} = \underline{2980 \text{ fps}}$

9.16 Eq. 9.2.4: $\Delta V = -\frac{\Delta p}{rc} = -\frac{\Delta p}{r\sqrt{kRT}} = -\frac{0.3}{.00237\sqrt{1.4 \times 1716 \times 519}} = \underline{-0.113 \text{ fps}}.$

Energy Eq: $\frac{V^2}{2} + c_p T = \frac{(V = \Delta V)^2}{2} + c_p(T + \Delta T)$. $\therefore 0 = V\Delta V + c_p \Delta T.$

$\therefore \Delta T = -c\Delta V \ / \ c_p = -\sqrt{1.4 \times 1716 \times 519}(-.113) \ / \ (0.24 \times 778 \times 32.2) = \underline{0.021^\circ \text{ F}}.$

Note: $c_p = .24 \frac{\text{Btu}}{\text{lbm -}^\circ \text{ F}} \times 778 \frac{\text{ft - lb}}{\text{Btu}} \times 32.2 \frac{\text{lbm}}{\text{slug}} = .24 \times 778 \times 32.2 \frac{\text{ft - lb}}{\text{slug -}^\circ \text{ F}}.$

Then $\frac{\text{ft}^2 \ / \sec^2}{\text{ft - lb / (slug -}^\circ \text{ F)}} = \frac{\text{ft}^2 - \text{lb - sec}^2 -^\circ \text{ F}}{\sec^2 - \text{ft - lb - ft}} = ^\circ \text{ F}.$ (units can be a pain!)

9.17 a) $rAV = rAV + rAdV + AVdr + AdrdV + rVdA + rdAdV + VdrdA + drdAdV$

Keep only the first order terms (the higher order terms—those with more than one differential quantity—will be negligible):

$$0 = rAdV + AVdr + rVdA$$

Divide by rAV:

$$\frac{dV}{V} + \frac{dr}{r} + \frac{dA}{A} = 0$$

b) Expand the r.h.s. of Eq. 9.3.5 (keep only first order terms):

$$\frac{V^2}{2} + \frac{k}{k-1}\frac{p}{r} = \frac{V^2 + 2VdV}{2} + \frac{k}{k-1}\frac{p+dp}{r+dr}.$$

Hence,

$$0 = \frac{2VdV}{2} + \frac{k}{k-1}\left(\frac{p+dp}{r+dr} - \frac{p}{r}\right)$$

$$= VdV + \frac{k}{k-1}\left(\frac{rp + rdp - pr - pdr}{r^2 + rdr}\right)$$

$$= VdV + \frac{k}{k-1}\left(\frac{rdp - pdr}{r^2}\right)$$

where we neglected rdr compared to r^2. For an isentropic process Eq. 9.2.8 gives $rdp = kpdr$, so the above becomes

$$0 = VdV + \frac{k}{k-1}\frac{kpdr - pdr}{r^2}$$

$$= VdV + \frac{k}{k-1}\frac{(k-1)pdr}{r^2} = VdV + k\frac{p}{r^2}dr$$

But $dr\ /\ r = -dV\ /\ V - dA\ /\ A$ so that the above equation is

$$0 = VdV + k\frac{p}{r}\left(-\frac{dV}{V} - \frac{dA}{A}\right)$$

which can be written as

$$\frac{dA}{A} = \left(\frac{V^2 r}{kp} - 1\right)\frac{dV}{V}.$$

Since $c^2 = kp\ /\ r$, and M = V/c, this is put in the form

$$\frac{dA}{A} = \left(\frac{V^2}{c^2} - 1\right)\frac{dV}{V} \quad \text{or} \quad \frac{dA}{A} = \left(\mathrm{M}^2 - 1\right)\frac{dV}{V}$$

c) Substituting in $V = \mathrm{M}c$, $c^2 = kRT$, and $R / c_p = (k-1)/k$, we find

$$\frac{T_0}{T} = \frac{V^2}{2c_p T} + 1 = \frac{\mathrm{M}^2 c^2}{2c_p T} + 1 = \frac{\mathrm{M}^2 kRT}{2c_p T} + 1$$

$$= \frac{\mathrm{M}^2 k(k-1)}{2k} + 1 = 1 + \frac{k-1}{2}\mathrm{M}^2.$$

d) $$\dot{m} = p\sqrt{\frac{k}{TR}}A\mathrm{M} = \frac{p_0\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{k/(1-k)}}{\sqrt{T_0}\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{-1/2}}\sqrt{\frac{k}{R}}A\mathrm{M}$$

$$= p_0\sqrt{\frac{k}{RT_0}}\mathrm{M}A\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{\frac{k+1}{2(1-k)}}$$

At the critical area A^*, $\mathrm{M}^* = 1$. Hence,

$$\dot{m} = p_0\sqrt{\frac{k}{RT_0}}A^*\left(\frac{k+1}{2}\right)^{\frac{k+1}{2(1-k)}}.$$

e) Since $\dot{m}$ is constant throughout the nozzle, we can equate Eq. 9.3.17 to Eq. 9.3.18:

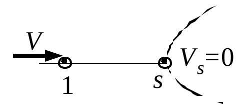
$$p_0\sqrt{\frac{k}{RT_0}}\mathrm{M}A\left(1 + \frac{k-1}{2}\mathrm{M}^2\right)^{\frac{k+1}{2(1-k)}} = p_0\sqrt{\frac{k}{RT_0}}A^*\left(\frac{k+1}{2}\right)^{\frac{k+1}{2(1-k)}}$$

or

$$\frac{A}{A^*} = \frac{1}{\mathrm{M}}\left[\frac{2 + (k-1)\mathrm{M}^2}{k+1}\right]^{\frac{k+1}{2(k-1)}}$$

9.18 a) $p_s = p_{atm} + 10 = 69.9 + 10 = 79.9$ kPa abs.

$p_1 = 69.9$ kPa abs.



From $1 \to s$: $\frac{V_1^2}{2} + \frac{p_1}{r_1} = \frac{p_s}{r_s}$. $r_s = r_1\left(\frac{p_s}{p_1}\right)^{1/k} = .906\left(\frac{79.9}{69.9}\right)^{1/1.4} = 0.997 \text{ kg / m}^3$.

$\therefore \frac{V_1^2}{2} + \frac{69\ 900}{.906} = \frac{79\ 900}{.997}$. $\therefore \underline{V_1 = 77.3 \text{ m / s}}$.

b) $p_s = 26.4 + 10 = 36.4$ kPa abs. $p_1 = 26.4$ kPa abs.

From $1 \to s$: $\frac{V_1^2}{2} + \frac{p_1}{r_1} = \frac{p_s}{r_s}$. $r_s = r_1\left(\frac{p_s}{p_1}\right)^{1/k} = 0.412\left(\frac{36.4}{26.4}\right)^{1/1.4} = 0.518 \text{ kg / m}^3$.

$\frac{V_1^2}{2} + \frac{26\ 400}{.412} = \frac{36\ 400}{.518}$. $\therefore \underline{V_1 = 111 \text{ m / s}}$.

9.19 a) $\frac{V_1^2}{2} + \frac{p_1}{r_1} = \frac{p_s}{r_s}$. $r_s = r_1\left(\frac{p_s}{p_1}\right)^{1/k} = 1.22\left(\frac{105}{101}\right)^{1/1.4} = 1.254 \text{ kg/m}^3$.

$\frac{V_1^2}{2} + \frac{1.4}{.4}\frac{101\ 000}{1.22} = \frac{105\ 000}{1.254}\frac{1.4}{.4}$. $\therefore V_1 = \underline{81.3 \text{ m/s}}$.

b) $\frac{V_1^2}{2} = \frac{4000}{1.22}$. $\therefore V_1 = \underline{81.0 \text{ m/s.}}$ % error $= \frac{81.3-81}{81.3} \times 100 = \underline{0.42\%}$.

9.20 Is $p_r < .5283 p_0$? $0.5283 \times 200 = 105.7$ kPa.

a) $p_r < .5283 p_0$. $\therefore$ choked flow. $\therefore \text{M}_e = 1$. $\therefore V_e^2 = kRT_e$. $p_e = 105.7$ kPa.

$1000 \times 298 = \frac{1.4 \times 287 T_e}{2} + 1000 T_e$. $\therefore T_e = 248.1$ K, $V_e = 315.8$ m / s.

$r_e = \frac{105.7}{.287 \times 248.1} = 1.484 \text{ kg / m}^3$. $\therefore \dot{m} = 1.484 \times p \times .01^2 \times 315.8 = \underline{0.1473 \text{ kg / s}}$.

b) $p_r > .5283 p_0$. $\therefore \text{M}_e < 1$. $1000 \times 298 = \frac{V_e^2}{2} + \frac{1.4}{.4}\frac{130\ 000}{r_e}$. $\frac{130}{200} = \left(\frac{r_e}{2.338}\right)^{1.4}$

$r_0 = \frac{200}{.287 \times 298} = 2.338$. $\therefore r_e = 1.7187 \text{ kg / m}^3$. $\therefore V_e = 257.9$ m / s.

$\therefore \dot{m} = 1.7187 \times p \times .01^2 \times 257.9 = \underline{0.1393 \text{ kg / s}}$.

9.21 Is $p_r < .5283 p_0$? $0.5283 \times 30 = 15.85$ psia.

a) $p_r < 15.85$. $\therefore$ choked flow and $\text{M}_e = 1$, $p_e = 15.85$ psia. $V_e^2 = kRT$.

$0.24 \times 530 = \frac{1.4 \times 1716 \times T_e}{2(778 \times 32.2)} + 0.24\ T_e$. $\therefore T_e = 441.7°$ R, $V_e = 1030$ fps.

$$r_e = \frac{15.85\times144}{1716\times441.7} = 0.003011 \text{ slug/ft}^3.$$

$$\therefore \dot m = .003011\times p\left(\frac{.5}{12}\right)^2\times1030 = \underline{0.01692 \text{ slug/sec}}.$$

b) $p_r >$ 15.85. $\therefore \text{M}_e < 1$, and $p_e = 20$ psia.

$$r_0 = \frac{30\times144}{1716\times530} = .00475 \text{ slug/ft}^3.$$

$$\therefore r_e = .00475\left(\frac{20}{30}\right)^{1/1.4} = .003556 \text{ slug/ft}^3.$$

$$0.24\times530(778\times32.2) = \frac{V_e^2}{2} + \frac{1.4}{.4}\frac{20\times144}{.003556}.$$

$$\therefore V_e = 838.9 \text{ fps.} \quad \therefore \dot m = .003556\times p\left(\frac{.5}{12}\right)^2\times838.9 = \underline{0.01627 \text{ slug / sec}}.$$

(Note: $c_p = 0.24 \text{ Btu/lbm-}^\circ\text{R} = 0.24\times778\frac{\text{ft-lb}}{\text{lbm-}^\circ\text{R}} = 0.24\times778\times32.2\frac{\text{ft-lb}}{\text{slug-}^\circ\text{R}}$.)

9.22 a) $p_r < .5283\, p_0$. $\therefore \text{M}_e = 1$. $\therefore p_e = .5283\times200 = 105.7$ kPa. $T_e = .8333\times298 = 248.3$ K.

$$r_e = \frac{105.7}{.287\times248.3} = 1.483 \text{ kg / m}^3. \quad V_e = \sqrt{1.4\times287\times248.3} = 315.9 \text{ m / s}.$$

$$\therefore \dot m = 1.483\times p\times.01^2\times315.9 = \underline{0.1472 \text{ kg / s}}.$$

b) $p_r > .5283\, p_0$. $\therefore p_e = 130$ kPa, $\frac{p_e}{p_0} = 0.65$. $\therefore \text{M}_e = .81$, $T_e = .884T_0$

$$r_e = \frac{130}{.287\times263.4} = 1.719 \text{ kg / m}^3, \quad V_e = .81\sqrt{1.4\times287\times263.4} = 263.5 \text{ m / s}.$$

$$\therefore \dot m = 1.719\times p\times.01^2\times263.5 = \underline{0.1423 \text{ kg / s}}.$$

9.23 a) $p_r < .5283\, p_0$. $\therefore \text{M}_e = 1$. $\therefore p_e = .5283\times30 = 15.85$ psia.

$$T_e = .8333\times530 = 441.6^\circ \text{R}.$$

$$\therefore r_e = \frac{15.85\times144}{1716\times441.6} = .003012 \frac{\text{slug}}{\text{ft}^3}. \quad V_e = \sqrt{1.4\times1716\times441.6} = 1030 \text{ fps}.$$

$$\dot m = .003012\times p\left(\frac{.5}{12}\right)^2\times1030 = \underline{0.01692 \text{ slug / sec}}.$$

b) $p_r > .5283\, p_0$. $\therefore p_e = 20$ psia. $\frac{p_e}{p_0} = \frac{20}{30} = .6667$. $\therefore \text{M}_e = .785$. $T_e = 0.890T_0$.

$$\therefore r_0 = \frac{20\times144}{1716\times472} = .00356. \quad V_e = .785\sqrt{1.4\times1716\times472} = 836 \text{ fps}.$$

$$\therefore \dot m = .00356 p\left(\frac{.5}{12}\right)^2\times836 = \underline{0.01664 \text{ slug / sec}}.$$

9.24 $p_e = .5283 \times 400 = \underline{211.3 \text{ kPa abs}}$. $T_e = .8333 \times 303 = 252.5$ K.

$V_e = \sqrt{1.4 \times 287 \times 252.5} = 318.5$ m / s. $\therefore \dot{m} = \dfrac{211.3}{.287 \times 252.5} \pi \times .05^2 \times 318.5 = \underline{7.29 \text{ kg / s}}$.

9.25 $p_e = .5283\, p_0 = 101$ kPa. $\therefore p_0 = \underline{191.2 \text{ kPa abs}}$. $T_e = .8333 \times 283 = 235.8$ K.

$V_e = \sqrt{1.4 \times 287 \times 235.8} = 307.8$ m / s. $\therefore \dot{m} = \dfrac{101}{.287 \times 235.8} \pi \times .03^2 \times 307.8 = \underline{1.30 \text{ kg / s}}$.

$p_0 = 2 \times 191.2 = 382.4$ kPa abs. $p_e = .5283\, p_0 = 202.0$ kPa abs. $T_e = 235.8$ K.

$V_e = 307.8$ m / s since $M_e = 1$. $\therefore \dot{m} = \dfrac{202}{.287 \times 235.8} \pi \times .03^2 \times 307.8 = \underline{2.60 \text{ kg / s}}$.

9.26 $p_e = .5283\, p_0 = 14.7$ psia. $\therefore p_0 = \underline{27.83 \text{ psia}}$. $T_e = .8333 \times 500 = 416.6^\circ$ R.

$V_e = \sqrt{1.4 \times 1716 \times 416.6} = 1000$ fps.

$\therefore \rho_e = 0.3203$ kg/m^3 and $p_e = \underline{199.4 \text{ kPa abs}}$.

$p_0 = 2 \times 27.83$. $p_e = 0.5283\, p_0 = 29.4$ psia, $T_e = 416.6^\circ$ R, $V_e = 1000$ fps.

$\therefore \underline{\dot{m} = 0.202 \text{ slug / sec}}$.

9.27 Treat the pipeline as a reservoir. Then, $p_e = .5283\, p_0 = 264.5$ kPa abs.

$M_e = 1$ and $V_e = \sqrt{1.4 \times 287 (.8333 \times 283)} = 307.8$ m / s.

$$\dot{m} = \frac{264.5}{.287 \times (.8333 \times 283)} \times 30 \times 10^{-4} \times 307.8 = 3.61 \text{ kg / s}.$$

$$\Delta V\!\!\!- = \frac{\dot{m}\Delta t}{\rho} = \frac{3.61 \times 6 \times 60}{264.5 / (.287 \times .8333 \times 283)} = \underline{333 \text{ m}^3}.$$

9.28 $5193 \times 300 = \dfrac{1.667 \times 2077\ T_e}{2} + 5193\ T_e$. $\therefore T_e = 225$ K. $\therefore p_e = 200 \left(\dfrac{225}{300}\right)^{\frac{1.667}{.667}}$

$= \underline{97.45 \text{ kPa}}$.

Next, $T_t = 225$ K, $p_t = 97.45$ kPa; $\therefore V_t = \sqrt{1.667 \times 2077 \times 225} = 882.6$ m / s.

$\rho_t = \dfrac{97.45}{2.077 \times 225} = 0.2085$ kg/m^3. $0.2085 \times \pi \times .03^2 \times 882.6 = \rho_e \pi \times 0.075^2 V_e$

$$5193 \times 300 = \frac{V_e^2}{2} + \frac{1.667}{.667}\frac{p_e}{\rho_e}. \quad p_e = 200\left(\frac{\rho_e}{200 / 2.077 \times 300}\right)^{1.667} = 1330 \rho_e^{1.667} \text{ kPa}.$$

$$= \frac{V_e^2}{2} + 3324 \times 10^3 \times 9.54 V_e^{-.667}.$$

or $3.116 \times 10^6 = V_e^2 + 63\ 420 \times 10^3 V_e^{-.667}$. Trial - and - error: $V_e = 91.8$ m / s.

$\therefore \rho_e = 0.3203$ kg/m^3 and $p_e = \underline{199.4 \text{ kPa abs}}$.

9.29 $\rho_1 = \frac{p_1}{RT_1} = \frac{300+100}{.287\times 293} = 4.757\ \text{kg}/\text{m}^3$. $\rho_2 = 4.757\left(\frac{340}{400}\right)^{1/1.4} = 4.236\ \text{kg}/\text{m}^3$.

$V_1 \times 4.757 \times 10^2 = V_2 \times 4.236 \times 5^2$. $\therefore V_2 = 4.492\ V_1$.

$\frac{V_1^2}{2} + \frac{k}{k-1}\frac{p_1}{\rho_1} = \frac{V_2^2}{2} + \frac{k}{k-1}\frac{p_2}{\rho_2}$. $\frac{V_1^2}{2} + \frac{1.4}{.4}\frac{400\ 000}{4.757} = \frac{4.492^2 V_1^2}{2} + \frac{1.4}{.4}\frac{340\ 000}{4.236}$.

$\therefore V_1 = 37.35\ \frac{\text{m}}{\text{s}}$.

$\therefore \dot{m} = \rho_1 A_1 V_1 = 4.757 \times \pi \times .05^2 \times 37.35 = \underline{1.395\ \text{kg}/\text{s}}$.

9.30 $\rho_1 = \frac{p_1}{RT_1} = \frac{(45+14.7)144}{1716\times 520} = 0.009634\ \frac{\text{slug}}{\text{ft}^3}$.

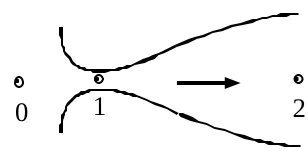
$\rho_2 = .009634\left(\frac{50.7}{59.7}\right)^{1/1.4} = .008573\ \text{slug}/\text{ft}^3$.

$V_1 \times .009634 \times 4^2 = V_2 \times .008573 \times 2^2$. $\therefore V_2 = 4.495\ V_1$.

$\frac{V_1^2}{2} + \frac{1.4}{.4}\frac{59.7\times 144}{.009634} = \frac{4.495^2 V_1^2}{2} + \frac{1.4}{.4}\frac{50.7\times 144}{.008573}$. $\therefore V_1 = 121.9$ fps.

$\therefore \dot{m} = .009634\pi \times (2/12)^2 \times 121.9 = \underline{0.1025\ \text{slug}/\text{sec}}$.

9.31 Energy $0 \to 2$: $1000 \times 303 = \frac{V_2^2}{2} + 1000\ T_2$. $V_2 = 3\sqrt{kRT_2}$



$\therefore p_{2\ell} = 1.627 \times 20 = 32.5$ kPa.

$\therefore p_2 = 200\left(\frac{107.9}{303}\right)^{\frac{1.4}{.4}} = 5.390$ kPa. $\rho_2 = \frac{5.39}{.287\times 107.9} = 0.1740\ \text{kg/m}^3$.

Energy $0 \to 1$: $1000 \times 303 = \frac{V_1^2}{2} + 1000\ \frac{V_1^2}{1.4\times 287}$. $\therefore V_1 = 318.4\ \text{m}/\text{s}$, $T_1 = 252.3$ K.

$p_1 = 200\left(\frac{252.3}{303}\right)^{\frac{1.4}{.4}} = 105.4$ kPa. $\rho_1 = \frac{105.4}{.287\times 252.3} = 1.455\ \text{kg}/\text{m}^3$.

Continuity: $1.455\pi \times .05^2 \times 318.4 = .174\pi\frac{d_2^2}{4} \times 3\sqrt{1.4\times 287\times 107.9}$. $\therefore \underline{d_2 = 0.2065\ \text{m}}$.

9.32 $V_t^2 = kRT_t$. $1000 \times 293 = \frac{1.4\times 287\ T_t}{2} + 1000\ T_t$. $\therefore T_t = 244.0$ K. $V_t = 313.1\ \text{m}/\text{s}$.

$\therefore p_t = 500\left(\frac{244}{293}\right)^{\frac{1.4}{.4}} = 263.5$ kPa abs. $\therefore \rho_t = \frac{263.5}{.287\times 244} = 3.763\ \text{kg}/\text{m}^3$.

$1000 \times 293 = \frac{V_e^2}{2} + \frac{1.4}{.4}\frac{p_e}{r_e}$. $3.763 \times p \times .025^2 \times 313.1 = r_e p \times .075^2 V_e$. $\frac{p_e}{r_e^{1.4}} = \frac{263\ 500}{3.763^{1.4}}$

$\therefore 293\ 000 = \frac{V_e^2}{2} + 1.014 \times 10^6 V_e^{-.4}$. Trial-and-error: $V_e = 22.2$ m/s, 659 m/s.

$\therefore r_e = 5.897,\ 0.1987\ \text{kg / m}^3$. $\therefore p_e =$ <u>494.2 kPa</u>, <u>4.29 kPa abs</u>.

9.33 $\frac{A_e}{A^*} = 9$. $\therefore \frac{p_e}{p_0} = 0.997$ from Table D.1. $\therefore p_e = 500 \times 997 =$ <u>498.5 kPa</u>.

and $\frac{p_e}{p_0} = 0.00855$ from Table D.1. $\therefore p_e =$ <u>4.28 kPa abs</u>.

9.34 $M_t = 1$. $\therefore p_t = .5283 \times 120 = 63.4$ psia, $T_t = .8333 \times 520 = 433.3^\circ$R.

$\therefore r_t = .01228\ \frac{\text{slug}}{\text{ft}^3}$. $\dot{m} = 1 = .01228 \frac{p\, d_t^2}{4}\sqrt{1.4 \times 1716 \times 433.3}$. $\therefore$ <u>$d_t = 0.319$ ft</u>.

$\frac{p}{p_0} = \frac{15}{120} = .125$. $\therefore M_e = 2.014$, $T_e = .552 \times 520 = 287^\circ$R, $V_e = 2.014\sqrt{1.4 \times 1716 \times 287}$

$=$ <u>684 fps</u>.

$\frac{A}{A^*} = 1.708$. $\therefore \frac{p\, d_e^2}{4} = 1.708 \frac{p \times .319^2}{4}$. $\therefore$ <u>$d_e = 0.417$ ft</u>.

9.35 $M_e = 4$. $\frac{A}{A^*} = 10.72$, $p_e = .006586 \times 2000 = 13.17$ kPa, $T_e = .2381 \times 293 = 69.76$ K.

For $\frac{A}{A^*} = 10.72$, $M_e = .0584$. $\therefore p_e = .9976\, p_0 = .9976 \times 2000 =$ <u>1995.2 kPa abs</u>.

9.36 Let $M_t = 1$. Neglect viscous effects. $M_1 = \frac{150}{\sqrt{1.4 \times 287 \times 303}} = 0.430$.

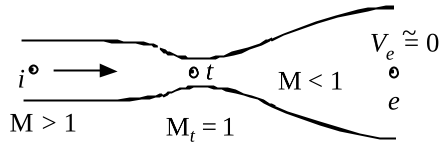
$\therefore \frac{A}{A^*} = 1.5007$. $\therefore A_t = \frac{A_1}{1.5007} = \frac{p \times .05^2}{1.5007} = \frac{p\, d_t^2}{4}$. $\therefore d_t = 0.0816$ m or <u>8.16 cm</u>.

9.37 $p_e = .5283 \times 400 = 211.3$ kPa abs. $T_{es} = .8333 \times 303 = 252.5$.

$.96 = \frac{303 - T_e}{303 - 252.5}$. $\therefore T_e = 254.5$ K. $\therefore V_e = \sqrt{1.4 \times 287 \times 254.5} = 319.8$ m/s.

$\therefore \dot{m} = \frac{211.3}{.287 \times 254.5} p \times .05^2 \times 319.8 =$ <u>7.27 kg/s</u>.

9.38 Isentropic flow. Since $k = 1.4$ for nitrogen, the isentropic table may be used.



$$M = 3: \quad \frac{A}{A^*} = 4.235.$$

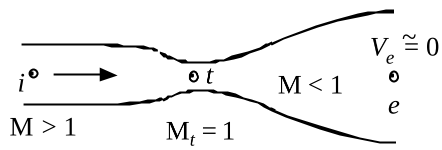
$$V_i = 3\sqrt{1.4 \times 297 \times 373} = 1181 \text{ m/s.} \quad r_i = \frac{100}{.297 \times 373} = .9027 \text{ kg/m}^3.$$

$$\therefore A_i = \frac{\dot{m}}{r_i V_i} = \frac{10}{.9027 \times 1181} = 0.00938 \text{ m}^2. \quad \therefore A_t = \frac{.00938}{4.235} = \underline{0.00221 \text{ m}^2}.$$

At $M = 3$, $T = .3571\ T_0$, $p = .02722\ p_0$.

$$\therefore T_0 = T_e = \frac{373}{.3571} = 1044 \text{ K or } \underline{772^\circ\text{C}}. \quad p_0 = \frac{100}{.02722} = p_e = \underline{3670 \text{ kPa}}.$$

9.39 Isentropic flow. Since $k = 1.4$ for nitrogen, the isentropic table may be used.



$$M = 3: \quad \frac{A}{A^*} = 4.235.$$

$$V_i = 3\sqrt{1.4 \times 1776 \times 660} = 3840 \text{ fps.} \quad r_i = \frac{15 \times 144}{1776 \times 660} = .001843 \ \frac{\text{slug}}{\text{ft}^3}.$$

$$A_i = \frac{.2}{.001843 \times 3840} = .0283 \text{ ft}^2. \quad \therefore A_t = \frac{.0283}{4.235} = \underline{0.00667 \text{ ft}^2}.$$

At $M = 3$, $T = .3571\ T_0$, $p = .02722\ p_0$.

$$\therefore T_0 = T_e = \frac{660}{.3571} = 1848^\circ\text{R or } \underline{1388^\circ\text{F}}. \quad p_0 = p_e = \frac{15}{.02722} = \underline{551 \text{ psia}}.$$

9.40 Assume $p_e = 101$ kPa. Then $r_e = \frac{101}{.189 \times 1273} = .4198 \text{ kg / m}^3$.

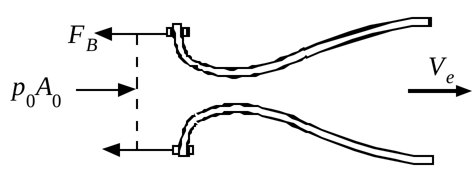
Momentum: $F = \dot{m}V = rAV^2$. $\frac{80\,000 \times 9.81}{6} = .4198p \times .25^2 V^2$.

$$\therefore \underline{V = 1260 \text{ m/s}}.$$

9.41 $F = \dot{m}V = rAV^2$. $r = \frac{101}{.287 \times 873} = .403 \text{ kg / m}^3$. (Assume gases are air.)

$100 \times 9.81 = .403 \times 200 \times 10^{-4} V^2$. $\therefore V = \underline{349 \text{ m/s}}$.

9.42 $M_t = 1$. $\frac{A_e}{A^*} = 4$; $\therefore M_e = 2.94$, $p_e = .02980\ p_0$.



$T_e = .3665\ T_0 = .3665 \times 300 = 109.95$ K,

$p_e = 100 = .0298\ p_0$. $\therefore p_0 = 3356$ kPa abs.

$$\therefore V_e = 2.94\sqrt{1.4 \times 287 \times 109.95} = 618 \text{ m / s.}$$

$$\therefore F_B = \frac{-100}{.287 \times 109.95} p \times .05^2 \times 618^2 + 3\ 356\ 000 p \times .2^2 = \underline{412\ 000 \text{ N}}.$$

9.43 Assume an isentropic flow; Eq. 9.3.13 provides

$$\frac{1.03p}{p} = \left(1 + \frac{k-1}{2}\text{M}^2\right)^{\frac{1}{k-1}}.$$

Using k = 1.4 this gives $\text{M}^2 = 0.0424$ or $\text{M} = 0.206$.

For standard conditions $V = \text{M}c = 0.206\sqrt{1.4 \times 287 \times 288} = \underline{70 \text{ m/s}}$.

9.44 a) $0.9850 \times 1000 = r_2 V_2$. $80\ 000 - p_2 = 0.985 \times 1000(V_2 - 1000)$

$$\frac{V_2^2 - 1000^2}{2} + \frac{1.4}{.4}\left(\frac{p_2}{r_2} - 287 \times 283\right) = 0. \quad \left(r_1 = \frac{80}{.287 \times 283} = .9850 \text{ kg / m}^3.\right)$$

$$\frac{V_2^2}{2} - \frac{1000^2}{2} + \frac{1.4}{.4}\frac{V_2}{985}(-985V_2 + 1\ 065\ 000) - 284\ 300 = 0$$

$$\therefore 3V_2^2 - 3784V_2 + 784\ 300 = 0. \quad \therefore V_2 = 261 \text{ m / s.} \quad r_2 = \underline{3.774 \text{ kg / m}^3}.$$

Substitute in and find $p_2 = \underline{808 \text{ kPa}}$.

$$\text{M}_1 = \frac{1000}{\sqrt{1.4 \times 287 \times 283}} = \underline{2.966}. \quad T_2 = \frac{808}{.287 \times 3.774} = 746 \text{ K or } \underline{473^\circ\text{C}}.$$

$$\text{M}_2 = \frac{261}{\sqrt{1.4 \times 287 \times 746}} = \underline{0.477}.$$

b) $\text{M}_1 = 1000 / \sqrt{1.4 \times 287 \times 283} = \underline{2.97}$. $\therefore \text{M}_2 = \underline{0.477}$. $p_2 = 10.12\, p_1 = \underline{809.6 \text{ kPa}}$.

$$T_2 = 2.644 \times 283 = 748 \text{ K or } \underline{475^\circ\text{C}}. \quad \therefore r_2 = \frac{809.6}{.287 \times 748} = \underline{3.771 \text{ kg / m}^3}.$$

9.45 a) $r_1 = \frac{12 \times 144}{1716 \times 500} = .002014 \ \frac{\text{slug}}{\text{ft}^3}$. $.002014 \times 3000 = r_2 V_2$.

Momentum: $12 \times 144 - p_2 = .002014 \times 3000(V_2 - 3000)$.

$$\frac{V_2^2 - 3000^2}{2} + \frac{1.4}{.4}\left(\frac{p_2}{r_2} - 1716 \times 500\right) = 0.$$

$$V_2^2 - 3000^2 + 7\left(\frac{V_2}{6.042}\right)(19{,}854 - 6.042V_2) - 6.006 \times 10^6 = 0.$$

$$\therefore 6V_2^2 - 23{,}000V_2 + 15 \times 10^6 = 0. \quad \therefore V_2 = 833 \text{ fps.} \quad r_2 = \underline{0.00725 \ \frac{\text{slug}}{\text{ft}^3}}.$$

$p_2 = \underline{102.9 \text{ psia}}$.

$$\text{M}_1 = \frac{3000}{\sqrt{1.4 \times 1716 \times 500}} = \underline{2.74}. \quad T_2 = \frac{102.9 \times 144}{1716 \times .00725} = 1191^\circ\text{R or } \underline{731^\circ\text{F}}.$$

$$M_2 = \frac{833}{\sqrt{1.4 \times 1716 \times 1191}} = \underline{0.492}.$$

b) $M_1 = 3000/\sqrt{1.4 \times 1716 \times 500} = \underline{2.74}$. $\therefore M_2 = \underline{0.493.}$ $p_2 = 8.592 \times 12 = \underline{103.1\text{ psia}}$.

$T_2 = 2.386 \times 500 = 1193^\circ\text{R}$ or $\underline{733^\circ\text{F}}$. $\therefore r_2 = \frac{103.1 \times 144}{1716 \times 1193} = \underline{0.00725\text{ slug / ft}^3}$.

9.46 $$\frac{r_2}{r_1} = \frac{p_2}{p_1}\frac{T_1}{T_2} = \frac{2kM_1^2 - k + 1}{k+1} \frac{(k+1)^2 M_1^2}{\left[1 + \frac{k-1}{2}M_1^2\right]\left[4kM_1^2 - 2k + 2\right]} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2}.$$

$M_1^2 = \frac{k+1}{2k}\frac{p_2}{p_1} + \frac{k-1}{2k}$. (This is Eq. 9.4.12). Substitute into above:

$$\frac{r_2}{r_1} = \frac{(k+1)\left[(k+1)\frac{p_2}{p_1} + (k-1)\right]}{4k + (k-1)\left\{(k+1)\frac{p_2}{p_1} + (k-1)\right\}} = \frac{(k+1)\left[(k+1)\frac{p_2}{p_1} + k - 1\right]}{(k+1)^2 + (k-1)(k+1)\frac{p_2}{p_1}}.$$

$$= \frac{k - 1 + (k+1)p_2 / p_1}{k + 1 + (k-1)p_2 / p_1}.$$

For a strong schock in which $\frac{p_2}{p_1} >> 1$, $\underline{\frac{r_2}{r_1} = \frac{k+1}{k-1}}$.

9.47 Assume standard conditions: $T_1 = 15^\circ\text{C}$, $r_1 = 101\text{ kPa}$.

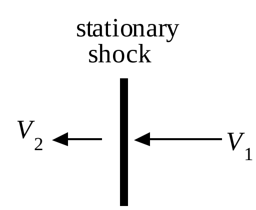
$\therefore V_1 = 2\sqrt{1.4 \times 287 \times 288} = 680\text{ m / s}$.

$M_1 = 2.$ $\therefore M_2 = .5774.$ $T_2 = 1.688 \times 288 = 486\text{ K}$.

$p_2 = 4.5 \times 101 = \underline{454\text{ kPa}}$.

$\therefore V_2 = .5774\sqrt{1.4 \times 287 \times 486} = 255\text{ m/s}$.

$\therefore V_{\text{induced}} = V_1 - V_2 = 680 - 255 = \underline{425\text{ m/s}}$.



The high pressure and high induced velocity cause extreme damage.

9.48 If $M_2 = .5$, then $M_1 = 2.645$. $\therefore V_1 = 2.645\sqrt{1.4 \times 287 \times 293} = \underline{908\text{ m / s}}$.

$p_2 = 8.00 \times 200 = \underline{1600\text{ kPa abs}}$. $r_2 = \frac{1600}{.287 \times (2.285 \times 293)} = \underline{8.33\text{ kg / m}^3}$.

9.49 If $M_2 = .5$, then $M_1 = 2.645$. $\therefore V_1 = 2.645\sqrt{1.4 \times 1716 \times 520} = \underline{1118\text{ fps}}$.

$p_2 = 8.00 \times 30 = \underline{240\text{ psia}}$. $r_2 = \frac{240 \times 144}{1716 \times (2.285 \times 520)} = \underline{0.01695\text{ slug / ft}^3}$.

9.50 $p_1 = .2615 \times 101 = 26.4$ kPa. $T_1 = 223.3$ K. $\mathrm{M}_1 = 1000 / \sqrt{1.4 \times 287 \times 223.3} = 3.34.$
$\therefore \mathrm{M}_2 = .4578.$ $p_2 = 12.85 \times 26.4 = 339$ kPa. $T_2 = 3.101 \times 223.3 = 692.5$ K.
For isentropic flow from ② → ⓪: For M = .458, p = .866 p_0 and
$T = .960\ T_0$. $\therefore p_0 = 339 / .866 = \underline{391 \text{ kPa abs}}$. $T_0 = 692.5 / .960 = 721$ K or $\underline{448^\circ \text{C}}$.

9.51 After the shock $\mathrm{M}_2 = .4752$, $p_2 = 10.33 \times 800 = 8264$ kPa abs.
For isentropic flow from ② → ⓪: For M = .475, p = .857 p_0.
$\therefore p_0 = 8264 / .857 = \underline{9640 \text{ kPa abs}}$.

9.52 $\dfrac{A}{A^*} = 4.$ $\therefore \mathrm{M}_e = .147.$ $p_e = .985\, p_0$ $\therefore p_0 = 101 / .985 = \underline{102.5 \text{ kPa abs}}$.
$\mathrm{M}_t = 1.$ $p_t = .5283 \times 102.5 = 54.15$ kPa. $T_t = .8333 \times 298 = 248.3$ K.
$\therefore \rho_t = \dfrac{54.15}{.287 \times 248.3} = .7599 \text{ kg/m}^3.$ $V_t = \sqrt{1.4 \times 287 \times 248.3} = 315.9$ m/s.
$\therefore \dot{m} = .7599 \times \pi \times .025^2 \times 315.9 = \underline{0.471 \text{ kg/s}}$. If throat area is reduced, M_t
remains at 1, $\rho_t = .7599 \text{ kg/m}^3$ and $\dot{m} = .7599 \times \pi \times .02^2 \times 315.9 = \underline{0.302 \text{ kg/s}}$.

9.53 $p_e = 101 \text{ kPa} = p_2.$ $\dfrac{A}{A^*} = 4.$ $\therefore \mathrm{M}_1 = 2.94$, and $p_2 / p_1 = 9.918.$
$\therefore p_1 = 101 / 9.918 = 10.18$ kPa. At $\mathrm{M}_1 = 2.94$, $p / p_0 = .0298.$
$\therefore p_0 = 10.18 / .0298 = \underline{342 \text{ kPa abs}}$.
$\mathrm{M}_t = 1$, $p_t = .5283 \times 342 = \underline{181 \text{ kPa abs}}$. $T_t = .8333 \times 293 = 244.1$ K.
$\therefore V_t = \sqrt{1.4 \times 287 \times 244.1} = \underline{313 \text{ m/s}}$.
$\mathrm{M}_1 = 2.94$, $p_1 = \underline{10.18 \text{ kPa abs}}$. $T_1 = .3665 \times 293 = 107.4$ K.
$\therefore V_1 = 2.94\sqrt{1.4 \times 287 \times 107.4} = \underline{611 \text{ m/s}}$.
$\mathrm{M}_2 = .4788$, $p_e = \underline{101 \text{ kPa}}$. $T_e = T_2 = 2.609 \times 107.4 = 280.2$ K.
$\therefore V_2 = .4788\sqrt{1.4 \times 287 \times 280.2} = \underline{161 \text{ m/s}}$.

9.54 $p_e = 14.7 \text{ psia} = p_2.$ $\dfrac{A}{A^*} = 4.$ $\therefore \mathrm{M}_1 = 2.94$, and $p_2 / p_1 = 9.918.$
$\therefore p_1 = 14.7 / 9.918 = 1.482$ psia. At $\mathrm{M}_1 = 2.94$, $p / p_0 = .0298.$
$\therefore p_0 = 1.482 / .0298 = \underline{49.7 \text{ psia}}$.
$\mathrm{M}_t = 1$, $p_t = .5283 \times 49.7 = \underline{26.3 \text{ psia}}$. $T_t = .8333 \times 520 = 433.3^\circ$R.
$\therefore V_t = \sqrt{1.4 \times 1716 \times 433.3} = \underline{1020 \text{ fps}}$.
$\mathrm{M}_1 = 2.94$, $p_1 = \underline{1.482 \text{ psia}}$. $T_1 = .3665 \times 520 = 190.6^\circ$R.
$\therefore V_1 = 2.94\sqrt{1.4 \times 1716 \times 190.6} = \underline{1989 \text{ fps}}$.

$M_2 = .4788,\ p_e = \underline{14.7\ \text{psia}}.\quad T_e = T_2 = 2.609 \times 190.6 = 497.3^\circ\ \text{R}.$

$\therefore V_2 = .4788\sqrt{1.4 \times 1716 \times 497.3} = \underline{523\ \text{fps}}.$

9.55 $M_t = 1.\ \ p_t = .5283 \times 500 = 264\ \text{kPa}.\ \ T_t = .8333 \times 298 = 248.3\ \text{K}.$

$\frac{A_1}{A^*} = \frac{8^2}{5^2} = 2.56.\quad \therefore M_1 = 2.47,\ p_1 = .0613 \times 500 = 30.65.$

$T_1 = .451 \times 298 = 134.4\ \text{K}.\quad \therefore V_1 = 2.47\sqrt{1.4 \times 287 \times 134.4} = \underline{574\ \text{m/s}}.$

$M_2 = .516,\ p_2 = 6.95 \times 30.65 = 213\ \text{kPa}.\quad T_2 = 2.108 \times 134.4 = 283.3\ \text{K}.$

After the shock it's isentropic flow. At $M = .516,\ \frac{A}{A^*} = 1.314.$

$p_{02} = .511 \times 500 = 255.5\ \text{kPa}.\ \ A^* = \frac{\pi \times .04^2}{1.314} = .003825\ \text{m}^2.$

$\frac{A_e}{A^*} = \frac{\pi \times .05^2}{.003825} = 2.05.\quad \therefore p_e = .940 \times 255.5 = \underline{240\ \text{kPa abs}}. = p_r.\ \ M_e = .298.$

$T_e = 283.3\left(\frac{213}{240}\right)^{.2857} = 273.8\ \text{K}.\quad \therefore V_e = .298\sqrt{1.4 \times 287 \times 273.8} = \underline{99\ \text{m/s}}.$

9.56 $p_t = .546\ p_0 = .546 \times 1200 = 655\ \text{kPa}.\ \ T_t = 673\left(\frac{655}{1200}\right)^{.3/1.3} = 585\ \text{K}.$

$\therefore \rho_t = \frac{655}{.462 \times 585} = 2.42\ \text{kg/m}^3.\ \ V_t = \sqrt{1.3 \times 462 \times 585} = 593\ \text{m/s}.\ \ (M_t = 1.)$

$\dot{m} = \rho_t A_t V_t.\quad \therefore 4 = 2.42 \times \frac{\pi \times d_t^2}{4} \times 593.\quad \therefore d_t = 0.060\ \text{m}\ \text{or}\ \underline{6\ \text{cm}}.$

$T_e = 673\left(\frac{101}{1200}\right)^{.3/1.3} = 380.2\ \text{K}\quad \therefore \rho_e = \frac{101}{.462 \times 380.2} = .575\ \text{kg/m}^3.$

$\frac{V_e^2}{2} + 1872 \times 380.2 = 1872 \times 673.\quad$ (Energy from ⓪ $\to$ e .) $(c_p = 1872\ \text{J/kg}\cdot\text{K})$

$\therefore V_e = 1050\ \text{m/s}.\quad \therefore 4 = .575\frac{\pi d_e^2}{4} \times 1050.\quad \therefore d_e = 0.092\ \text{m}\ \text{or}\ \underline{9.2\ \text{cm}}.$

9.57 $M_e = 1.\ \ p_e = .546\ p_0 = .546 \times 1000 = 546\ \text{kPa}.$

$T_e = 623\left(\frac{546}{1000}\right)^{\frac{.3}{1.3}} = 542\ \text{K}.\quad \therefore \rho_e = \frac{546}{.462 \times 542} = 2.18\ \frac{\text{kg}}{\text{m}^3}.$

$V_e = \sqrt{1.3 \times 462 \times 542} = 571\ \text{m/s}.\quad 15 = 2.18\frac{\pi d_e^2}{4} \times 571.\quad \therefore d_e = 0.124\ \text{m or}\ \underline{12.4\ \text{cm}}.$

9.58 $\quad M_e = 1.\ \ p_e = .546 \times 150 = 81.9 \text{ psia.}\ \ T_e = 1160\left(\frac{81.9}{150}\right)^{\frac{.3}{1.3}} = 1009^\circ \text{R.}$

$\therefore r_e = \frac{81.9 \times 144}{2762 \times 1009} = .00423\ \frac{\text{slug}}{\text{ft}^3}.\ \ V_e = \sqrt{1.3 \times 2760 \times 1009} = 1903 \text{ fps.}$

$.25 = .00423 \frac{p d_e^2}{4} \times 1903.\quad \therefore d_e = 0.199 \text{ ft. or } \underline{2.39''}.$

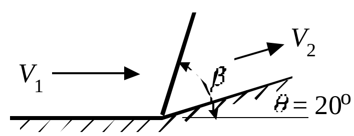
9.59 $\quad M_t = 1.\ \ p_t = .546 \times 1200 = 655 \text{ kPa.}\ \ T_t = 673\left(\frac{655}{1200}\right)^{.3/1.3} = 585 \text{ K.}$

$\therefore V_t = \sqrt{1.3 \times 462 \times 585} = 593 \text{ m/s.}\ \ r_t = \frac{655}{.462 \times 585} = 2.42 \text{ kg/m}^3.$

$\therefore \dot{m} = 2.42 \times p \times .0075^2 \times 593 = \underline{0.254 \text{ kg/s per nozzle}}$

$T_e = 673\left(\frac{120}{1200}\right)^{.3/1.3} = \underline{396 \text{ K}}.$

9.60 $\quad M_1 = \frac{800}{\sqrt{1.4 \times 287 \times 303}} = 2.29.$



From Fig. 9.15, $b = 46^\circ,\ \ 79^\circ.$

a) $b = 46^\circ.\ \ \therefore M_{1n} = 2.29 \sin 46^\circ = 1.65.$

$\therefore M_{2n} = .654 = M_2 \sin(46^\circ - 20^\circ).\quad \therefore \underline{M_2 = 1.49}.$

$p_2 = 3.01 \times 40 = \underline{120.4 \text{ kPa abs}}.\ \ T_2 = 1.423 \times 303 = 431 \text{ K.}$

$V_2 = \sqrt{1.4 \times 287 \times 431} \times 1.49 = \underline{620 \text{ m/s}}.$

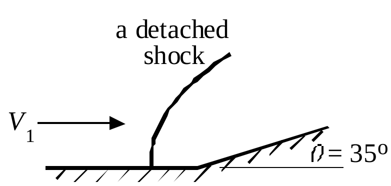
b) $b = 79^\circ.\ \ \therefore M_{1n} = 2.29 \sin 79^\circ = 2.25.\ \ \therefore M_{2n} = .541 = M_2 \sin(79^\circ - 20^\circ).$

$\therefore \underline{M_2 = 0.631}.$

$p_2 = 5.74 \times 40 = \underline{230 \text{ kPa abs}}.\ \ T_2 = 1.90 \times 303 = 576 \text{ K.}$

$V_2 = \sqrt{1.4 \times 287 \times 576} \times .631 = \underline{303 \text{ m/s}}.$

c)



9.61 $\beta_1 = 40^\circ$. $\quad \therefore \theta = 10^\circ$.

$\mathrm{M}_{1n} = 2\sin 40^\circ = 1.29. \quad \therefore \mathrm{M}_{2n} = .791 = \mathrm{M}_2 \sin(40^\circ - 10^\circ). \quad \therefore \mathrm{M}_2 = 1.58.$

If $\theta_2 = 10^\circ$ then, with $\mathrm{M} = 1.58$, $\beta_2 = 51^\circ$. $1.58\sin 51^\circ = \mathrm{M}_{2n}$. $\therefore \mathrm{M}_{2n} = 1.23$.

$\therefore \mathrm{M}_{3n} = .824 = \mathrm{M}_3 \sin(51^\circ - 10^\circ). \quad \therefore \underline{\mathrm{M}_3 = 1.26}. \quad \beta = \beta_2 - 10 = 51 - 10 = \underline{41^\circ}.$

9.62 $\mathrm{M}_{1n} = 3.5\sin 35^\circ = 2.01. \quad \therefore \mathrm{M}_{2n} = .576. \quad T_2 = 1.696 \times 303 = 514\text{ K}.$

$$\mathrm{M}_2 = \frac{.576}{\sin(35^\circ - 20^\circ)} = 2.26. \quad \theta_1 = 20^\circ = \theta_2. \quad \therefore \beta_2 = 47^\circ.$$

$\mathrm{M}_{2n} = 2.26\sin 47^\circ = 1.65. \quad \therefore \mathrm{M}_{3n} = .654 = \mathrm{M}_3 \sin(47^\circ - 20^\circ). \quad \therefore \mathrm{M}_3 = 1.44.$

$T_3 = 1.423 \times 514 = 731\text{ K}. \quad V_3 = \mathrm{M}_3\sqrt{kRT_3} = 1.44\sqrt{1.4 \times 287 \times 731} = \underline{780\text{ m/s}}.$

9.63 $\mathrm{M}_{1n} = 3.5\sin 35^\circ = 2.01. \quad \therefore \mathrm{M}_{2n} = .576. \quad T_2 = 1.696 \times 490 = 831^\circ\text{R}.$

$$\mathrm{M}_2 = \frac{.576}{\sin(35^\circ - 20^\circ)} = 2.26. \quad \theta_1 = 20^\circ = \theta_2. \quad \therefore \beta_2 = 47^\circ.$$

$\mathrm{M}_{2n} = 2.26\sin 47^\circ = 1.65. \quad \therefore \mathrm{M}_{3n} = .654 = \mathrm{M}_3 \sin(47^\circ - 20^\circ). \quad \therefore \mathrm{M}_3 = 1.44.$

$T_3 = 1.423 \times 831 = 1180^\circ\text{R}. \quad V_3 = \mathrm{M}_3\sqrt{kRT_3} = 1.44\sqrt{1.4 \times 1716 \times 1180} = \underline{2420\text{ fps}}.$

9.64 $\mathrm{M}_1 = 3,\ \theta = 10^\circ. \quad \therefore \beta_1 = 28^\circ. \quad \mathrm{M}_{1n} = 3\sin 28^\circ = 1.41. \quad \therefore \mathrm{M}_{2n} = .736.$

$\therefore p_2 = 2.153 \times 40 = 86.1\text{ kPa}.$

$$\mathrm{M}_2 = \frac{.736}{\sin(28^\circ - 10^\circ)} = 2.38. \quad \therefore p_3 = 6.442 \times 86.1 = \underline{555\text{ kPa}}.$$

$(p_3)_{\text{normal}} = 10.33 \times 40 = \underline{413\text{ kPa}}.$

9.65 At $\mathrm{M}_1 = 3$, $\theta_1 = 49.8^\circ$, $\mu_1 = 19.47^\circ$. (See Fig. 9.18.)

$\theta_1 + \theta_2 = 49.8 + 25 = 74.8^\circ. \quad \therefore \mathrm{M}_2 = 4.78.$

From isentropic flow table: $p_2 = p_1 \dfrac{p_0}{p_1}\dfrac{p_2}{p_0} = 20 \times \dfrac{1}{.02722} \times .002452 = \underline{1.80\text{ kPa}}.$

$$T_2 = T_1 \frac{T_0}{T_1}\frac{T_2}{T_0} = 253 \times \frac{1}{.3571} \times .1795 = 127\text{K or } \underline{-146^\circ\text{C}}. \quad \mu_2 = 12.08^\circ.$$

$V_2 = 4.78\sqrt{1.4 \times 287 \times 127} = \underline{1080\text{ m/s}}. \quad \alpha = 90 + 25 - 70.53 - 12.08 = \underline{32.4^\circ}.$

9.66 $\theta_1 = 26.4^\circ$. For $\mathrm{M} = 4$, $\theta = 65.8^\circ$. (See Fig. 9.18.)

$\therefore \theta = 65.8 - 26.4 = \underline{39.4^\circ}.$

$$T_2 = T_1 \frac{T_0}{T_1}\frac{T_2}{T_0} = 273\frac{1}{.5556} \times .2381 = 117\text{ K}. \quad \therefore V_2 = 4\sqrt{1.4 \times 287 \times 117} = \underline{867\text{ m/s}}.$$

$T_2 = \underline{-156^\circ\text{C}}.$

9.67 $q = 26.4^\circ$. For $M = 4$, $q = 65.8^\circ$. $\therefore q = 65.8 - 26.4 = \underline{39.4^\circ}$.

$$T_2 = T_1 \frac{T_0}{T_1}\frac{T_2}{T_0} = 490\frac{1}{.5556}\times .2381 = 210^\circ \text{R or } \underline{-250^\circ\text{F}}.$$

$$V_2 = 4\sqrt{1.4\times 1716\times 210} = \underline{2840 \text{ fps}}.$$

9.68 a) $q_1 = 39.1^\circ$. $q_2 = 39.1 + 5 = 44.1^\circ$. $\therefore M_u = 2.72$. $p_{2u} = 20\frac{1}{.0585}\times .04165$

$$= \underline{14.24 \text{ kPa}}.$$

For $q = 5^\circ$ and $M = 2.5$, $b = 27^\circ$. $M_{1n} = 2.5\sin 27^\circ = 1.13$. $\therefore M_{2n} = .889$.

$$\therefore p_{2\ell} = 1.32\times 20 = \underline{26.4 \text{ kPa}}.$$

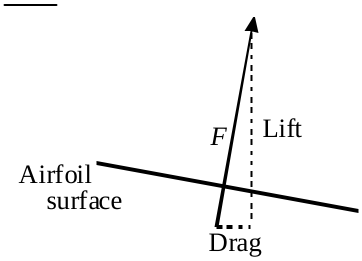
$$M_\ell = M_2 = \frac{.889}{\sin(27^\circ - 5^\circ)} = 2.37.$$

b) $M = 2.72$, $q = 5^\circ$. $\therefore b = 25^\circ$. $M_{1n} = 2.72\sin 25^\circ = 1.15$, $M_{2n} = .875$.

$$M_{2u} = \frac{.875}{\sin(25^\circ - 5^\circ)} = \underline{2.56}.$$

For $M = 2.37$, $q = 36.0^\circ$. For $q = 36 + 5 = 41^\circ$, $M_{2\ell} = \underline{2.58}$.

c) Force on plate $= (26.4 - 14.24)\times 1000\times A = F$.



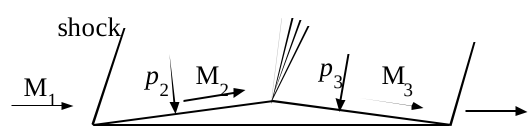
$$C_L = \frac{F\cos 5^\circ}{\frac{1}{2}r_1 V_1^2 A} = \frac{12.2\times .996\times 1000A}{\frac{1}{2}\times 1.4\times 2.5^2\times 20\ 000A} = \underline{0.139}.$$

d) $$C_D = \frac{F\sin 5^\circ}{\frac{1}{2}r_1 V_1^2 A} = \frac{12.2\times 1000A\times .0872}{\frac{1}{2}\times 1.4\times 2.5^2\times 20\ 000A} = \underline{0.0122}.$$

9.69 $b = 19^\circ$. $M_{1n} = 4\sin 19^\circ = 1.30$. $\therefore p_2 = 1.805\times 20 = 36.1$ kPa. $M_{2n} = .786$.

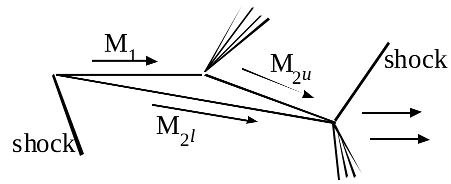
$$M_2 = \frac{.786}{\sin(19^\circ - 5^\circ)} = 3.25. \quad q_1 = 54.36. \quad q_2 = 59.36. \quad \therefore M_3 = 3.55.$$

$$p_3 = p_2\frac{p_0}{p_2}\frac{p_3}{p_0} = 36.1\times\frac{1}{.0188}\times .0122 = 23.4 \text{ kPa}.$$



$$C_D = \frac{\left(36.1\frac{A}{2} - 23.4\frac{A}{2}\right)\sin 5^\circ}{\frac{1}{2}rV_1^2 A} = \frac{6.35\times .0872}{\frac{1}{2}\times 1.4\times 4^2\times 20} = \underline{0.0025}.$$

9.70 If $q = 5^\circ$ with $M_1 = 4,$ then Fig. 9.15 $\rightarrow b = 18^\circ$.



$M_{1n} = 4\sin 18^\circ = 1.24. \quad \therefore M_{2n} = .818.$

$\therefore p_{2\ell} = 1.627 \times 20 = 32.5$ kPa.

$$\therefore M_{2\ell} = \frac{.818}{\sin(18^\circ - 5^\circ)} = 3.64.$$

At $M_1 = 4,\ q_1 = 65.8^\circ$. At 75.8° $M_{2u} = 4.88$. $p_{2u} = p_1 \dfrac{p_0}{p}\dfrac{p_2}{p_0} = 20\dfrac{.002177}{.006586}$

$= 6.61$ kPa.

$$C_L = \frac{\text{Lift}}{\frac{1}{2} r V_1^2 A} = \frac{32.5A\cos 5^\circ - 20 \times A/2 - 6.61 \times A/2 \times \cos 10^\circ}{\frac{1}{2} \times 1.4 \times 4^2 \times 20A} = \underline{0.0854}.$$

$$C_D = \frac{\text{Drag}}{\frac{1}{2} r V_1^2 A} = \frac{32.5A\sin 5^\circ - 6.61 \times A/2 \times \sin 10^\circ}{\frac{1}{2} \times 1.4 \times 4^2 \times 20A} = \underline{0.010}.$$